

# Comment on 'Uniqueness of the Equation for Quantum State Vector Collapse'

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A diffusive stochastic Schrödinger equation (SSE) is shown for the first time, such that contributes to a non-completely positive dynamics. This contradicts to a recent Letter claiming that SSEs, under most general conditions, enforce complete-positivity. The general form and parametrization of the SSE in the Letter is different from an alternative simpler result, the difference is shown to be completely redundant because of the gauge-freedom of the state vector's phase.

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A recent Letter [1] investigated markovian stochastic Schrödinger equations (SSEs) under the assumption of no-faster-than-light signalling [2]. I found that Theorem 1, claiming that the evolution of the density matrix  $\rho$  must be completely-positive (CP), is incorrect. Theorem 2 constructs the most general diffusive SSE for the wave function  $\psi$ , which looks different from the simpler results in Ref. [3]. I prove that the difference is redundant.

If Theorem 1 were true, no markovian SSE would exist for the non-CP qubit master equation [4]:

$$\frac{d\rho}{dt} = \sum_{k=1}^3 c_k (\sigma_k \rho \sigma_k - \rho), \quad c_1 = c_2 = -c_3 = 1. \quad (1)$$

I consider the following SSE (cf. [5] for a jump process):

$$d\psi = -\frac{1}{2} \sum_{k=1}^3 c_k (\sigma_k - n_k)^2 \psi dt + \sqrt{2} n_z \psi_{\perp} dW \quad (2)$$

where  $n_k = \langle \psi | \sigma_k | \psi \rangle$  and  $\psi_{\perp}$  is orthogonal to  $\psi$ , we can express it by  $\psi_{\perp} = (1 - n_z^2)^{-1/2} (n_y \sigma_x - n_x \sigma_y) \psi$ . The SSE (2) yields the master equation (1) for  $\rho = \mathbb{E} |\psi\rangle \langle \psi|$ . The proof goes like this. From Eq. (2) we get

$$\frac{d\rho}{dt} = -\frac{1}{2} \mathbb{E} \sum_{k=1}^3 c_k \left\{ (\sigma_k - n_k)^2, |\psi\rangle \langle \psi| \right\} + 2 \mathbb{E} n_z^2 |\psi_{\perp}\rangle \langle \psi_{\perp}|. \quad (3)$$

One can confirm the identity

$$2n_z^2 |\psi_{\perp}\rangle \langle \psi_{\perp}| = \sum_{k=1}^3 c_k (\sigma_k - n_k) |\psi\rangle \langle \psi| (\sigma_k - n_k) \quad (4)$$

which, when inserted into (3), leads to the linear master equation (1). Hence, Theorem 1 cannot be correct. The proof fails clearly if the number  $n$  of independent Lindblad operators  $L_k$  is bigger than the dimension  $d$  [6].

For CP master equations, the Letter's Theorem 2 is correct. The authors mention that Ref. [3] had answered the same question but the Letter does not compare the results. I remedy the omission. An additional gauge transformation  $\psi \rightarrow \exp(-id\chi)\psi$  with phase

$d\chi = \text{Im} \sum_k \langle \psi | L_k^{(\psi)} | \psi \rangle (\ell_k^{(\psi)} dt + dW_k)$  brings the Letter's SSE (4) to the form

$$d\psi = \left[ -iHdt + \sum_{k=1}^N \sum_{j=1}^n u_{kj}^{(\psi)} (L_j - \langle L_j \rangle) dW_k - \frac{1}{2} \sum_{k=1}^n (L_k^{\dagger} L_k - 2 \langle L_k \rangle^* L_k + |\langle L_k \rangle|^2) dt \right] \psi \quad (5)$$

where  $\langle L_k \rangle = \langle \psi | L_k | \psi \rangle$ . The matrix  $u$  has gone from the drift part! The resulting SSE coincides exactly with Eq. (8.1) in Ref. [3], implying the following relationship between the noises of [3] and the Letter, respectively:

$$d\xi_j^* = \sum_{k=1}^N dW_k u_{kj}, \quad j = 1, 2, \dots, n \leq N. \quad (6)$$

In Ref. [3], all physically different SSEs are uniquely parametrized by the  $n \times n$  complex symmetric correlation matrices  $s_{jl} = (\mathbb{E} d\xi_j d\xi_l^*)/dt$  (to avoid confusion, here we use  $s$  for  $u$  of (4.1) in [3]). Now Eq. (6) establishes the correspondence between the  $u$  and  $s$ :

$$s_{jl}^* = \sum_{k=1}^N u_{kj} u_{kl}, \quad j, l = 1, 2, \dots, n \leq N. \quad (7)$$

As I said, the matrix  $s_{jl}$ , only constrained by  $\|s\|$ , cf. (4.3) in [3], is in one-to-one correspondence with the physically different SSEs at a given CP-evolution of  $\rho$ . The matrix  $u_{kj}$  is not, its part  $N \geq j > n$  is redundant. Now (7) shows a further redundancy: both  $u$  and  $Ou$ , with any  $N \times N$  orthogonal matrix  $O$ , yield the same SSE.

Ref. [3] derived the SSEs under CP master equation from standard quantum monitoring. The SSE (2) is the first diffusive SSE considered ever that underlies non-CP master equation, its physical relevance, if any, needs further studies.

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[6] The proof is false for any number  $n > 1$ , it would be good to know which further natural conditions might render the theorem true ( private communication from the authors of [1]).